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RKKY INDIRECT EXCHANGE IN LOW-DIMENSIONAL SUPERCONDUCTORS

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The indirect exchange interaction between localized moments (LMs), mediated by one- and two-dimensional superconducting electron gas, is calculated by means of thermodynamic Green functions. The interaction potential is expressed in terms of higher transcendental functions of distance R between LMs. The asymptotic behavior at large R is presented for 1D and 2D cases. It is shown that an additional long range potential appears in the superconducting state as compared with the normal one.

1. INTRODUCTION

THE RECENTLY discovered high-temperature superconductors (HTSC) have very anisotropic physical properties. For example, in $YBa_2Cu_3O_7$-structure compounds the almost two-dimensional conductivity is accomplished by copper–oxygen planes nearby the Y ions or the rare earth ions, which substitute Y in the structure unit. The observation of Gd antiferromagnetic ordering in $GdBa_2Cu_3O_7$ with a Néel temperature $T_N = 2.24\,\mathrm{K}$ [1], which is considerably lower than the superconducting transition temperature T_c, attracts the attention to investigate magnetic interactions mediated by *two-dimensional* (*in general, by low-dimensional*) *superconducting* electrons.

The Hamiltonian of exchange interaction between two localized spins $\mathbf{S}_1$ and $\mathbf{S}_2$, placed at a distance R between them, can be written as

$$\mathscr{H} = -\mathbf{J}(R)\mathbf{S}_1\mathbf{S}_2. \tag{1}$$

For the RKKY interaction based on the local exchange between a localized spin $\mathbf{S}_1$ and a conduction electron spin σ_i

$$\mathscr{H}^i_{sf} = -\frac{J_{sf}}{N}\mathbf{S}_i\sigma_i. \tag{2}$$

The interaction potential $\mathbf{J}(R)$ in equation (1) can be expressed via thermodynamic Green functions (see, e.g., [2], where the RKKY exchange in three-dimensional superconductor has been calculated):

$$\mathbf{J}(R) = -\frac{J_{sf}^2}{2N^2}T\sum_n\{G_\omega(\mathbf{R})G_\omega(-\mathbf{R}) + F_\omega(\mathbf{R})F_\omega(-\mathbf{R})\}. \tag{3}$$

Here $G_\omega(\mathbf{R})$ and $F_\omega(\mathbf{R})$ are the "normal" and "anomalous" Green functions of a superconductor, respectively, J_{sf} is the coupling constant of conduction electrons and localized spins, $\omega = \pi T(2n + 1)$ is the Matsubara fermion frequency, $n = 0, \pm 1, \pm 2, \dots$, and N is the number of atoms in a volume. Green functions can be calculated as Fourier-transforms of standard momentum dependent Green functions of a superconductor [3]:

$$G_\omega(\mathbf{R}) = \int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}G_\omega(\mathbf{p})\,\mathrm{e}^{i\mathbf{pR}};$$
$$G_\omega(\mathbf{p}) = -\frac{i\omega + \xi_\mathbf{p}}{\omega^2 + \Delta^2 + \xi_\mathbf{p}^2}, \tag{4}$$
$$F_\omega(\mathbf{p}) = \frac{\Delta}{\omega^2 + \Delta^2 + \xi_\mathbf{p}^2},$$

where $\xi_\mathbf{p} = \varepsilon_\mathbf{p} - \varepsilon_F$ is the electron band energy with respect to the Fermi energy ε_F, $\Delta \equiv \Delta(T)$ is the gap in the excitation spectrum of a superconductor. Hereafter, we assume the band to be parabolic: $\varepsilon_\mathbf{p} = p^2/2m$ with an effective electron mass m, and begin specific calculations in the one-dimensional (1D) case.

2. 1D-SUPERCONDUCTOR

First of all, we convert the one-dimensional integral (4) to the positive half-axis and change variable from p to ξ by the approximation

$$\xi = (p^2 - p_F^2)/2m \simeq v_F(p - p_F), \tag{5}$$

where p_F and v_F are the Fermi momentum and velocity, respectively. Then closing the integration path on the upper half-plane for the positive-sign exponent and on the lower half-plane for the